

Colliding Bubble Worlds

Warren B. Perkins*

*Department of Physics,
University of Wales Swansea,
Singleton Park, Swansea, SA2 8PP,
Wales*

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Abstract

We consider a cosmological model in which our Universe is a spherically symmetric bubble wall in 5-dimensional anti-de Sitter spacetime. We argue that the bubble on which we live will undergo collisions with other similar bubbles and estimate the spectrum of such collisions. The collision rate is found to be independent of the age of our Universe. Collisions with small bubbles provide an experimental signature of this scenario, while collisions with larger bubbles would be catastrophic.

1 Introduction

There has been a great deal of interest recently in higher dimension theories with non-compact extra dimensions [1,2]. Rather than compactify the extra dimensions to leave the three observed large spatial dimensions, matter is localised to a 3-brane. While gravity exists in the bulk, there is a gravitational zero-mode on the brane which mimics four dimensional gravity at low energy. Our low energy world thus appears to be four dimensional.

While many particle physics constraints on such models have been addressed in the context of *plane* branes [1,2], it is also possible for these branes to be closed, expanding domain walls [3,4,5]. These two pictures are closely related [6]. Both plane branes and bubbles can be created, complete with their 5-dimensional anti-de Sitter (AdS_5) bulk spacetimes, from nothing, by the appropriate instantons [7,8,9].

In this paper we consider an alternative cosmological view in which our brane is an expanding bubble nucleated in a pre-existing AdS_5 bulk. In the spirit of more conventional bubble nucleation settings, we consider the bubble to separate two regions of AdS_5 with differing cosmological constants. The effective Friedmann equation on our brane is considered in section 2. In this scenario, our bubble is simply one of many bubbles forming in the AdS_5 bulk. In section 3 we consider the spectrum of bubble-bubble collisions. While providing possible experimental evidence for our scenario, there is also the possibility of a catastrophic bubble-bubble collision in our neighbourhood.

*w.perkins@swansea.ac.uk

2 Effective Friedmann Equation On The Bubble

We consider a bubble separating two AdS_5 regions with different cosmological constants, Λ_{\pm} . The effective Friedmann equation in this situation has been analysed in ref.5, while the linearized gravity on such branes is discussed in ref.10. Here, in order to specify our model, we review the results of ref.5.

The metrics on either side of the bubble have the form,

$$ds^2 = -(k + H_{\pm}^2 A^2) L_{\pm}(t) dT^2 + (k + H_{\pm}^2 A^2)^{-1} dA^2 + A^2 [d\chi^2 + f_k^2(\chi)(d\theta^2 + \sin^2(\theta)d\phi^2)], \quad (1)$$

where,

$$H_{\pm}^2 = -\frac{\Lambda_{\pm}}{6}, \quad (2)$$

$f_{-1}(\chi) = \sinh \chi$, $f_0(\chi) = 1$ and $f_1(\chi) = \sin \chi$. $L_{\pm}(t)$ are lapse functions that allow the coordinates to be matched at the bubble. For simplicity we set $L_+(t) = 1$.

If we let the position of the bubble be $T = t(\tau)$, $A = a(\tau)$, where τ is the proper time on the bubble, the induced metric on the bubble is,

$$ds_4^2 = -d\tau^2 + a(\tau)^2 [d\chi^2 + f_k^2(\chi)(d\theta^2 + \sin^2(\theta)d\phi^2)]. \quad (3)$$

As the bulk cosmological constants differ on either side of the bubble, the Israel matching condition [11] is used to relate the change in the extrinsic curvature at the bubble to the energy-momentum of the bubble. Assuming a perfect fluid form for the stress-energy tensor on the bubble, $T_{\nu}^{\mu} = \text{diag}(-\rho, p, p, p)$, the effective Friedmann equation on the bubble is found to be [5],

$$\frac{8}{3}\pi G \rho a = \sqrt{\dot{a}^2 + k - \frac{\Lambda_+}{6}a^2} - \sqrt{\dot{a}^2 + k - \frac{\Lambda_-}{6}a^2}. \quad (4)$$

This can be cast into a more familiar form by splitting the bubble stress-energy into a constant tension part and a matter contribution,

$$\rho = \rho_m + \sigma. \quad (5)$$

At late times, $\rho_m \ll \sigma$ and it is possible to expand the right hand side of the effective Friedmann equation as a power series in ρ ,

$$H^2 + \frac{k}{R^2} = \Lambda_{\text{eff.}} + \Phi \rho_m + \Psi \rho_m^2 + \dots \quad (6)$$

The effective cosmological constant can be set to zero by tuning the bubble tension, σ , according to the constraint,

$$\left(\frac{8}{3}\pi G \sigma\right)^2 = \left[\sqrt{\frac{-\Lambda_+}{6}} \pm \sqrt{\frac{-\Lambda_-}{6}}\right]^2. \quad (7)$$

The coefficients Φ and Ψ are then given by,

$$\Phi = \frac{4}{3}\pi G \left[4 \frac{\sqrt{\frac{\Lambda_+ \Lambda_-}{6}}}{\left| \sqrt{\frac{-\Lambda_+}{6}} \pm \sqrt{\frac{-\Lambda_-}{6}} \right|} \right], \quad \Psi = \frac{16}{9}\pi^2 G^2 \left[1 + 3 \frac{(\sqrt{-\Lambda_+} \mp \sqrt{-\Lambda_-})^2}{[\sqrt{-\Lambda_+} \pm \sqrt{-\Lambda_-}]^2} \right], \quad (8)$$

where the overall sign ambiguity in taking the square root of σ^2 has been resolved by requiring that both σ and Φ are positive. There remain two possible values for the bubble tension,

$$\sigma_+ = \frac{3}{8\pi G} \left[\sqrt{\frac{-\Lambda_+}{6}} + \sqrt{\frac{-\Lambda_-}{6}} \right], \quad \sigma_- = \frac{3}{8\pi G} \left| \sqrt{\frac{-\Lambda_+}{6}} - \sqrt{\frac{-\Lambda_-}{6}} \right|. \quad (9)$$

The latter solution is discussed in ref. 5.

In either case, at late times, we have a Friedmannian evolution of the scale factor on our bubble. As usual in these scenarios, there is also a non-zero coefficient of ρ_m^2 and the effective four dimensional Planck scale is determined by the fundamental Planck scale and the bulk cosmological constants.

3 Bubble-Bubble collisions

If we live on an expanding bubble that was nucleated in AdS_5 , we must consider the possibility of other bubbles forming in the same manner and colliding with ours. The nucleation process in the special case, $\Lambda_+ = \Lambda_-$, has been considered [12]. However, without a concrete underlying model, we cannot determine the tunnelling probability or bubble nucleation rate, but we can estimate the spectrum of bubble-bubble collisions.

We assume that the nucleation rate per unit volume, Γ , is constant, so that the probability of a nucleation event in the region $T \rightarrow T + dT$, $A \rightarrow A + dA$, $\chi \rightarrow \chi + d\chi$, $\theta \rightarrow \theta + d\theta$, $\phi \rightarrow \phi + d\phi$ is $\Gamma A^3 f_k^2(\chi) \sin(\theta) dT dA d\chi d\theta d\phi$. If we only consider small bubbles colliding with a large bubble, we are only interested in nucleation events in the vicinity of the large bubble. These have nucleation probability per unit time per unit bubble volume of approximately Γ .

The evolution of a single bubble in the AdS_5 has been determined in ref.5. In terms of the bubble proper time τ , the position of the shell is given by,

$$A = a(\tau), \quad T = t(\tau), \quad (10)$$

where $a(\tau)$ is determined by the modified Friedmann equation and,

$$\frac{dt}{d\tau} = \frac{\sqrt{k + H^2 a^2 + \left(\frac{da}{d\tau}\right)^2}}{k + H^2 a^2}. \quad (11)$$

For simplicity we consider the $k = 1$ case with the standard power law approximations for the evolution of the scale factor,

$$a = \alpha \tau^p, \quad p = \frac{1}{4}, \frac{1}{2}, \frac{2}{3}. \quad (12)$$

The last two values of p correspond to the standard radiation and matter dominated eras respectively. The first value holds at early times when the bubble is radiation dominated and the ρ_m^2 term in the effective Friedmann equation is dominant. Using (11), the background metric time at the bubble can then be related to the scale factor. At early times, $\left(\frac{da}{d\tau}\right)^2 \gg 1 + H^2 a^2$, leading to,

$$t - t_0 \simeq a, \quad (13)$$

while at late times, $H^2 a^2 \gg 1 + (\frac{da}{dt})^2$, we find,

$$t - t_0 \sim \frac{1}{H\alpha} \frac{\tau^{1-p}}{1-p} \rightarrow a \sim \alpha[(1-p)H\alpha(t-t_0)]^{\frac{p}{1-p}} = \begin{cases} \frac{H\alpha^2}{2}(t-t_0) & p = \frac{1}{2} \\ \frac{H^2\alpha^3}{9}(t-t_0)^2 & p = \frac{2}{3} \end{cases} \quad (14)$$

While the coordinate speed of the bubble seems to grow at late times, the physical speed of the bubble wall relative to the bubble centre is close to unity at early times but drops like $(t-t_0)^{-3}$ at late times.

In order to determine the evolution of a pair of bubbles, we take two such bubbles with different centres. The centre of the second bubble is at the origin of a shifted coordinate system defined by,

$$\begin{aligned} \tan H\tilde{T} &= \cosh p \tan HT - \sinh p \frac{\cos \chi}{\cos HT} \frac{AH}{\sqrt{1+A^2H^2}}, \\ \tilde{A}^2 &= A^2 \sin^2 \chi + [\sinh p \sin HT \sqrt{\frac{1}{H^2} + A^2} - A \cosh p \cos \chi]^2, \\ \cos \tilde{\chi} &= \frac{-\sinh p \sin HT \sqrt{\frac{1}{H^2} + A^2} + \cosh p A \cos \chi}{[A^2 \sin^2 \chi + (\sinh p \sin HT \sqrt{\frac{1}{H^2} + A^2} - A \cosh p \cos \chi)^2]^{1/2}}, \\ \tilde{\theta} &= \theta, \quad \tilde{\phi} = \phi. \end{aligned} \quad (15)$$

The metric in these coordinates is precisely that given in (1). The origin of this system lies on the $\sin \chi = 0$ line, while the origin of the original system lies on the $\sin \tilde{\chi} = 0$ line. The initial point of contact of the two bubble walls also lies on these lines, so we can work with the simplified transformations,

$$\begin{aligned} \tan H\tilde{T} &= \cosh p \tan HT - \frac{\sinh p}{\cos HT} \frac{AH}{\sqrt{1+A^2H^2}}, \\ \tilde{A} &= \sinh p \sin HT \sqrt{\frac{1}{H^2} + A^2} - \cosh p A. \end{aligned} \quad (16)$$

If a bubble nucleates at $\tilde{A} = 0$, we can use (10) and (6) to determine its evolution in the $(\tilde{T}, \tilde{A}, \tilde{\chi}, \tilde{\theta}, \tilde{\phi})$ coordinates. We assume that the centre of this bubble is at rest with respect to the centre of the first bubble at nucleation. If this nucleation event occurs at $T = T_2$, $\tilde{T} = \tilde{T}_2$, we have, $HT_2 = H\tilde{T}_2 = \pi/2$ and the spacing of the centres is given by,

$$H\tilde{A}_0 = \sinh p. \quad (17)$$

We consider a case relevant to our current Universe: one old bubble ($p = 2/3$) colliding with a younger bubble ($p = 1/2$). The old bubble has its origin at $\tilde{A} = 0$ and it is large, so the second bubble must nucleate a long way from $\tilde{A} = 0$. Thus, $H\tilde{A}_0$ is large and from (17) we have $\sinh p \sim \cosh p$. For simplicity we assume that the time between the nucleation of the second bubble and the collision of the two bubbles is short compared to H^{-1} . Denoting the elapsed times by ΔT and $\Delta \tilde{T}$, we can use (16) to relate the two at the position of the second bubble.

To leading order we find,

$$(H\Delta\tilde{T})^2 = \frac{(H\Delta T)^2}{\cosh^2 p [\sin HT - \frac{AH}{\sqrt{1+A^2H^2}}]^2}, \quad (18)$$

where A and T are the coordinates of the bubble nucleated at $A = 0$. The bubble spacing is given by,

$$\Delta\tilde{A} \sim \tilde{A}_0 [\sin HT \sqrt{1+A^2H^2} - AH] - \frac{H^2\alpha^3}{9} (\tilde{T}_2 - \tilde{T}_1)^2, \quad (19)$$

where the first term represents the position of the young bubble wall and the second is the radius of the old bubble. Assuming that $AH \ll 1$ and using $A \sim H\alpha^2\Delta T/2$ for the young bubble, we find to leading order in ΔT and $\Delta\tilde{T}$,

$$\begin{aligned} \Delta\tilde{A} &\sim \tilde{A}_0 [1 - \frac{1}{2}H^2\alpha^2\Delta T] - \frac{H^2\alpha^3}{9} [(\tilde{T} - \tilde{T}_1) + 2\Delta\tilde{T}(\tilde{T}_2 - \tilde{T}_1)] \\ &\sim \Delta\tilde{A}_0 - \Delta\tilde{T} \left\{ \frac{\cosh p}{2} \tilde{A}_0 H^2\alpha^2 + 2\frac{H^2\alpha^3}{9} (\tilde{T}_2 - \tilde{T}_1) \right\}. \end{aligned} \quad (20)$$

The elapsed time before collision is then,

$$\delta\tilde{T} \propto \frac{\Delta\tilde{A}_0}{\tilde{A}_0^2}. \quad (21)$$

In the case of young bubbles colliding with an old one, \tilde{A}_0 is roughly the scale factor of the old bubble, so we take it to be constant for all young bubbles hitting at an instant.

We denote the number of bubbles impacting with radius between \tilde{R}_{im} and $\tilde{R}_{im} + \delta\tilde{R}_{im}$ in a time interval \tilde{T} to $\tilde{T} + \delta\tilde{T}$ by,

$$\gamma(\tilde{R}_{im}, \tilde{T}) \delta\tilde{R}_{im} \delta\tilde{T}. \quad (22)$$

The size of the small bubble on impact is determined by the initial distance between the large bubble and the small bubble nucleation site. Thus bubbles with impact radius between \tilde{R}_{im} and $\tilde{R}_{im} + \delta\tilde{R}_{im}$ were nucleated in a region of width $\delta\tilde{R}_{im}$. The spread in nucleation time is isimilarly the spread in arrival time. As the nucleation probability per unit space-time volume is constant, we have,

$$\gamma(\tilde{R}_{im}, \tilde{T}) \sim \gamma(\tilde{T}) \sim \text{const.} \quad (23)$$

We are interested in the collision rate observed by the bubble dwellers. For the old bubble we have the usual matter dominated evolution of the scale factor, $\tilde{A}_0 \sim \tilde{\tau}^{2/3}$, so the coordinate time and bubble proper time are related by,

$$\frac{d\tilde{T}}{d\tilde{\tau}} \propto \tilde{\tau}^{-2/3} \quad (24)$$

In terms of the old bubble proper time, $\tilde{\tau}$, the collsion rate is,

$$\gamma(\tilde{R}_{im}, \tilde{\tau}) \propto \tilde{\tau}^{-2/3}. \quad (25)$$

If the scale factor of the smaller bubble at impact is a_{im} , we have, $a_{im} \sim \tilde{R}_{im}/H\tilde{A}_0$, leading to,

$$\gamma(a_{im}, \tilde{\tau}) \propto \sim \text{const.} \quad (26)$$

From the point of view of an observer at rest with respect to the centre of the old bubble, the slowly moving old bubble wall is bombarded a constant flux of smaller bubbles which are expanding at speeds of order unity.

We assume that the collision of another, smaller bubble with our own manifests itself as an injection of energy into some region. If the energy injected into the old bubble is proportional to a_{im}^q , the energy spectrum of impacts is given by,

$$\gamma(E, \tilde{\tau}) \propto E^{-1+1/q}. \quad (27)$$

The spectral index depends on q , but is always greater than -1.

According to the bubble dwellers, there is a constant rate of collisions with young bubbles. The smallest bubbles give the largest impact rate in a given energy interval and thus provide the most likely experimental signature of such a model. At the opposite end of the scale we have bubbles with ages upto fractions of a megayear (as measured in their own proper times) and sizes upto 10^{-3} of our own bubble Universe. Collisions with such bubbles would be catastrophic.

4 Conclusions

We have considered a brane-world scenario in which our brane is just one of many bubbles nucleated in an AdS_5 bulk. The late time evolution of the scale factor is Friedmannian. A signature of this particular scenario is the random energy injections arising from bubble-bubble collisions. In the $k = 1$ case, the rate at which small bubbles collide with our own is estimated to vary approximately as E^p , where E is the energy contained in the impacting bubble and $p > -1$.

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